

# Устойчивое горение в токамаке с пикированными профилями

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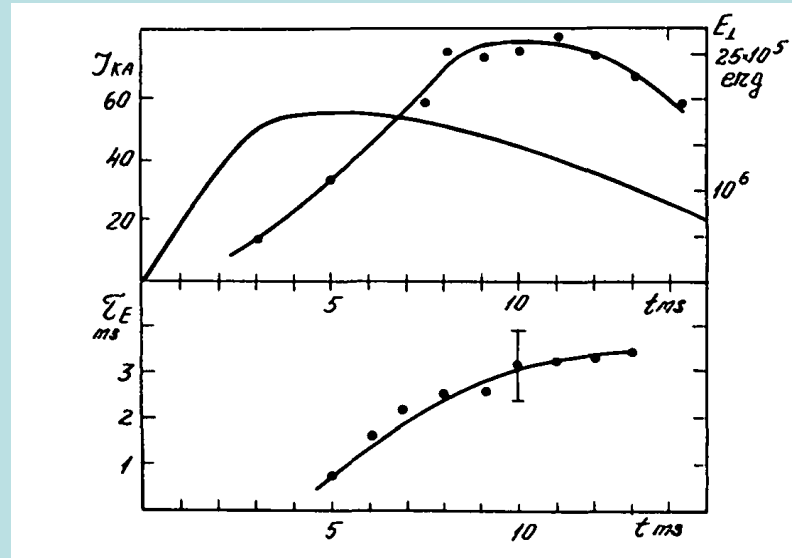
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**Плазма токамака с каноническими профилями в L-моде может гореть устойчиво на скиновых временах, поскольку тепловыделение растет как температура в квадрате, а сток тепла на границе быстрее, как температура в четвертой степени. Требуемые параметры близки к строящемуся токамаку SPARC, но профили должны быть более пикированы и сферичны. Степень сферичности определяется компромиссом инженерной сложности и выигрышем от улучшения удержания.**

**Методы усиления пикированности можно и нужно заложить в проект следующего российского токамака.**

# Улучшение удержания сбросом тока

T-3 Утроение времени удержания! S. Mirnov Nuclear Fusion, 9, (1969) 57



Я узнал о результате Мирнова 1969 в 2021 после рассказов о моих предсказаниях 1994-2003. Большинство последующих рекордов использовали сброс тока.

**Плазма сама собирается в центр токамака за турбулентные времена! Ксения Разумова 1994, устное сообщение**

Парадокс самособирание плазмы в центре, это причина успеха токамаков и ключ к пониманию и улучшению токамаков.

# Устойчивое горение Солнца и его турбулентный аттрактор

- Горит только в центре Солнца и энергия переносится к поверхности всплывающей горячей плазмой. Турбулентный аттрактор определяется сохранением вдоль траектории удельной энтропии  **$s=const$**  (адиабата). Как следствие, постоянная энтропия устанавливается почти во всем объеме, подобно концентрации сахара в кофе.
- Профиль Аттрактора турбулентного равнораспределения с точностью 99 процентов совпадает с экспериментом по данным сейсмоки
- Тепловой поток излучается с поверхности как  $T^4$
- В токамаке будет похоже, только инвариант другой

# Турбулентный аттрактор токамака выведен из инварианта $nv=const$

- Парадокс Разумовой сужает поиск инварианта, определяющего турбулентный аттрактор. Мои блуждания с бесконечным числом инвариантов в замороженности в уравнении Власова окончились простым признанием:

Плазма заморожена в полоидальное магнитное поле  $nv=const$

где  $v$  – удельный полоидальный объем

- $nv=const$  в токамаках  $v$  пропорционально  $q$ , с предположением одномерной адиабаты  $T=n^2$  это немедленно ведет к пикированности давления  $nv^3=const$  и всем свойствам канонических профилей, внутренним барьерам и объяснению парадоксов Мирнова и Разумовой.
- В. Coppi, *Nonclassical Transport and the "Principle of Profile Consistency"*, Comments Plasma Phys. Cont. Fusion **5**, 261 (1980)
- Yu. Esipchuk and K. Razumova Plasma Phys. Cont. Fusion **28**, 1253 (1986)
- Граничные условия Пастухова, Смирнова, Голдстоуна со стоком энергии  $T^4$ , как на Солнце, дополняют аттрактор.

# Comparison of Baker-Rosenbluth formula with experimental data

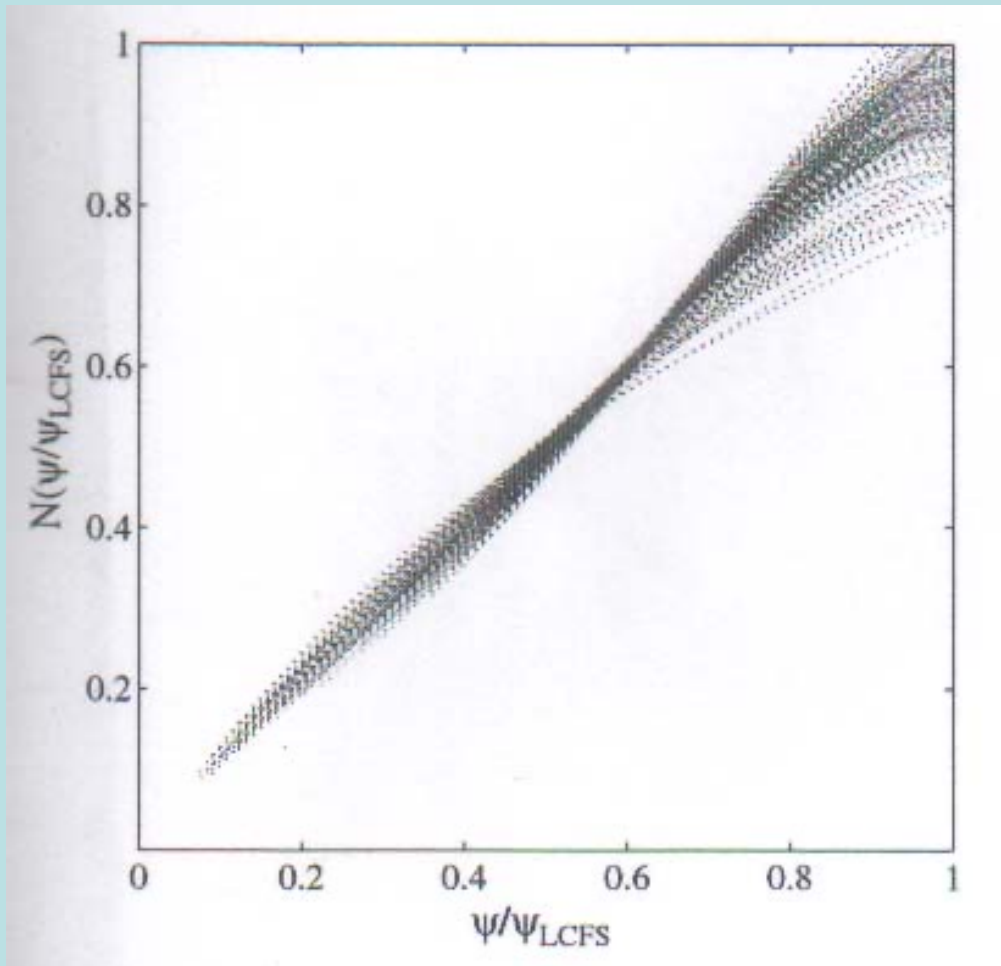
- The mechanism of particle peaking and TEP were popularized by A. Gruzinov, M. Isichenko and P. Diamond Phys. Plasmas **2**, 3541 (1995) and M. Isichenko, A. Gruzinov, P. Diamond and P. Yushmanov Phys. Plasmas **3**, 1916 (1996).
- D. Baker and M. Rosenbluth Phys. Plasmas **5**, 2936 (1998) generalized the V. Yankov JETP Letters, **60**, 71 (1994) formula

$$n \sim 1/\nu$$

by introduction of a fitting parameter alpha (in large aspect ratio tokamaks  $\nu$  is proportional to  $q$ )

$$n(r) = 1/q^\alpha(r)$$

- The Baker and Rosenbluth approximation has been experimentally tested in TCV tokamak with variable magnetic geometry, Weisen et al Nuclear Fusion **42**, 136 (2002)



Weisen coordinates.

Integrated normalized particle content versus normalized poloidal flux in an OH TCV data set. 226 density profiles from Weisen et al. (2002)

Best fit is  $\alpha = 1.0$  not a 0.9 or a 1.1

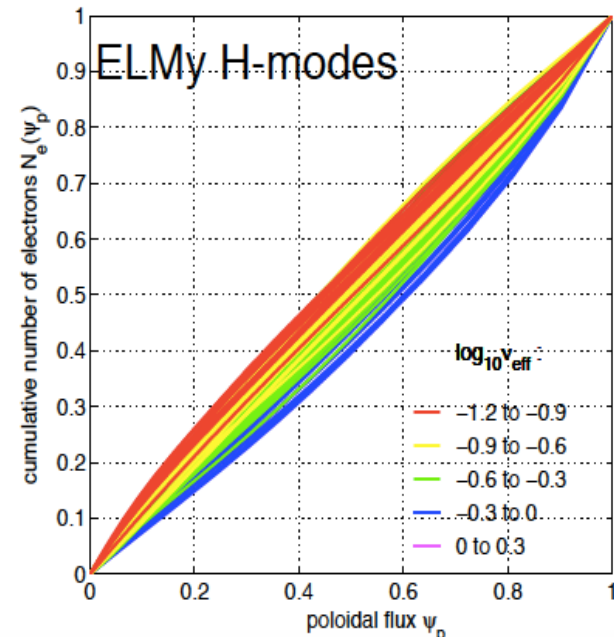
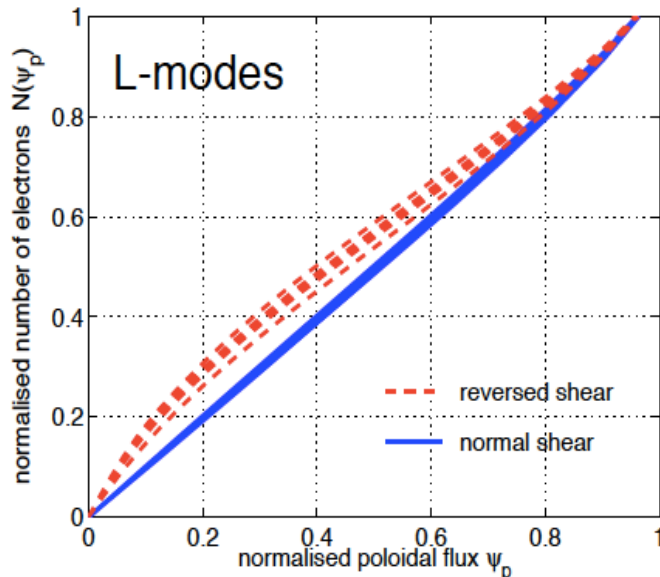
# JET density profiles H. Weisen *et al* poster EX/P6-31 20th IAEA FEC, Vilamoura, Portugal, 1-6 Nov. 2004 JET L-mode, Reversed shear, H-mode and TEP

## Shear dependence in qualitative agreement with Turbulent EquiPartition / curvature pinch

Due to invariance of  $\mu$  and  $J$ , turbulent diffusion causes trapped electrons to spread over poloidal flux ( $dN_e/d\Psi = \text{const}$ ):

$$\int_a^b n_e dV = N_e \Big|_a^b \propto |\psi(b) - \psi(a)|$$

corresponding to  $C_q=1$  ( &  $C_\varepsilon=0$ ). Fair agreement with L-modes (normal shear) and low  $V_{\text{eff}}$  H-modes



## Theory / interpretational issues: the verses don't quite rime...

- TEP/ curvature pinch not supposed to apply at high  $v_{\text{eff}}$  ??? but no  $v_{\text{eff}}$  dependence in L-mode.
- Density profiles not hollow in negative shear region ??? What causes peaking there? If thermodiffusion, why is there no direct evidence ?
- In TEP picture L-mode and low  $v_{\text{eff}}$  H-modes behave as if all electrons were trapped. ??? Why should all electrons behave like trapped ones?
- Max peaking higher than expected from TEP restricted to trapped electrons (Isichenko 1996, Baker 1998) or fluid theory (Garbet 2003) ??? but no evidence for other contributions
- Curvature pinch dominates in ITG regime ??? But then, why no shear dependence at  $v_{\text{eff}} > 0.2$ ?

JET team discussion, **23 знака ?** На одном слайде.

23 вопроса сводятся к одному: ПОЧЕМУ профили  $n \sim 1/v$  возникают везде, хотя американские работы их не предсказывали?

Инварианты - это стальные рельсы анализа.

Турбулентный аттрактор токамака выведен из единственного фундаментального инварианта, имеет предсказательную силу, и теория входит в эксперимент, как патрон в патронник.



# Топор есть, надо рубить

Improvement of confinement by a weakening the poloidal magnetic field at the boundary, Yankov JETP Letters 2003

Улучшение удержания – это естественное следствие аттрактора  $n \sim 1/\nu$ , поскольку увеличение  $\nu$  влечет расширение плазмы у границы и более низкую плотность и температуру.

Большее  $\nu$  можно достигнуть:

1. Сбросом тока. Хорошо для зажигания++
2. Увеличение числа X точек+? 3,4, or 5?
3. Сферический токамак+?
4. Расположение X точек дальше от оси. Отрицательная треугольность!+
5. Большая вытянутость+? 0.5-0.78

# Параметры токамака с пикированными профилями и возможным зажиганием, далекая экстраполяция, как и ITER но из теории

- **Мой вариант оптимизации оказался близок к SPARC, который должны запустить в 2025м, поэтому часть параметров просто возьму со SPARC:**

[https://en.wikipedia.org/wiki/SPARC\\_\(tokamak\)](https://en.wikipedia.org/wiki/SPARC_(tokamak))

- Большой радиус 1.85 м
- Магнитное поле 12.2 Т Лента Superox! Мы выиграли от прогресса ВТСП.
- Нагрев 2 МВт
- Полезен Li

## Теперь отличия:

- **Главная разница в пикированности, отношении величины в центре к величине внутри сепаратрисы, для трех переменных:  $1/v$  25, n 25, T 125**

**Обычно пикированность по плотности меньше десяти.**

- Малый радиус предлагаю 1 м вместо 0.57 м, что труднее
- Ток всего 4 МА вместо 8.7 МА, что легче
- Длительность нагрева в несколько раз короче, что легче
- Нужен сброс тока на треть от максимума 4 МА, требует усилий
- Нужен ЭЦР для управления профилем тока, в том числе и для внутренних барьеров

# Выводы

**Методы усиления пикированности можно и нужно заложить в проект следующего российского токамака**

**Теория неканонических профилей, построенная на инварианте  $n\nu=\text{const}$ , имеет предсказательную силу. Для уточнений нужна обработка старых экспериментов, особенно сферических и крутых токамаков. Нужны расчеты неканонических профилей и условий зажигания.**

**Плазма в токамаке с неканоническими профилями в L-моде может гореть устойчиво на скиновых временах.**

**Требуемые параметры близки к строящемуся токамаку SPARC, но профили должны быть более пикированы и сферичны.**

**Степень сферичности определяется компромиссом инженерной сложности и выигрышем от улучшения удержания.**

**Неоценимую помощь в поиске и анализе экспериментов оказали К. Разумова, С. Мирнов, С. Шарапов, В. Пастухов, П. Юшманов, В. Вершков, Ю. Днестровский, J. J. Rasmussen, J. Nycander, D. Baker, M. Rosenbluth, H. Weisen и многие другие.**

# Appendix

# Подготовка инструментов

- Законы сохранения появились до уравнений движения и надежней уравнений движения. Связка ключей.
- Инвариант Poincare определен в 3+3+1 фазовом объеме и означает отсутствие вихревых сил в Гамильтоновых переменных.

$$I = \oint \vec{p} d\vec{q}$$

- Если импульс  $\vec{p}$  есть функция координат  $\vec{q}$ , то 3+3+1 инвариант Пуанкаре можно спроектировать в обычное 3+1 пространство.
- Если момент включает механическую и магнитную части, то возникает сохранение обобщенной завихренности

$$\vec{p} = m\vec{v} + e\vec{A}/c$$

- В воде хранят механическую часть, а в плазме только магнитное поле.
- Если траектории почти периодичны, то появляются адиабатические инварианты. Если навиваются на торы, то КАМ инварианты.
- Я обобщил понятие замороженности на уравнение Власова, включая дрейфовые формы, УФН 97.
- Учебники выводят эти инварианты как независимые, что непорядок.

Tokamak TEPs are derived from the frozen-in law, while the frozen-n law is derived from the Poincare Invariant

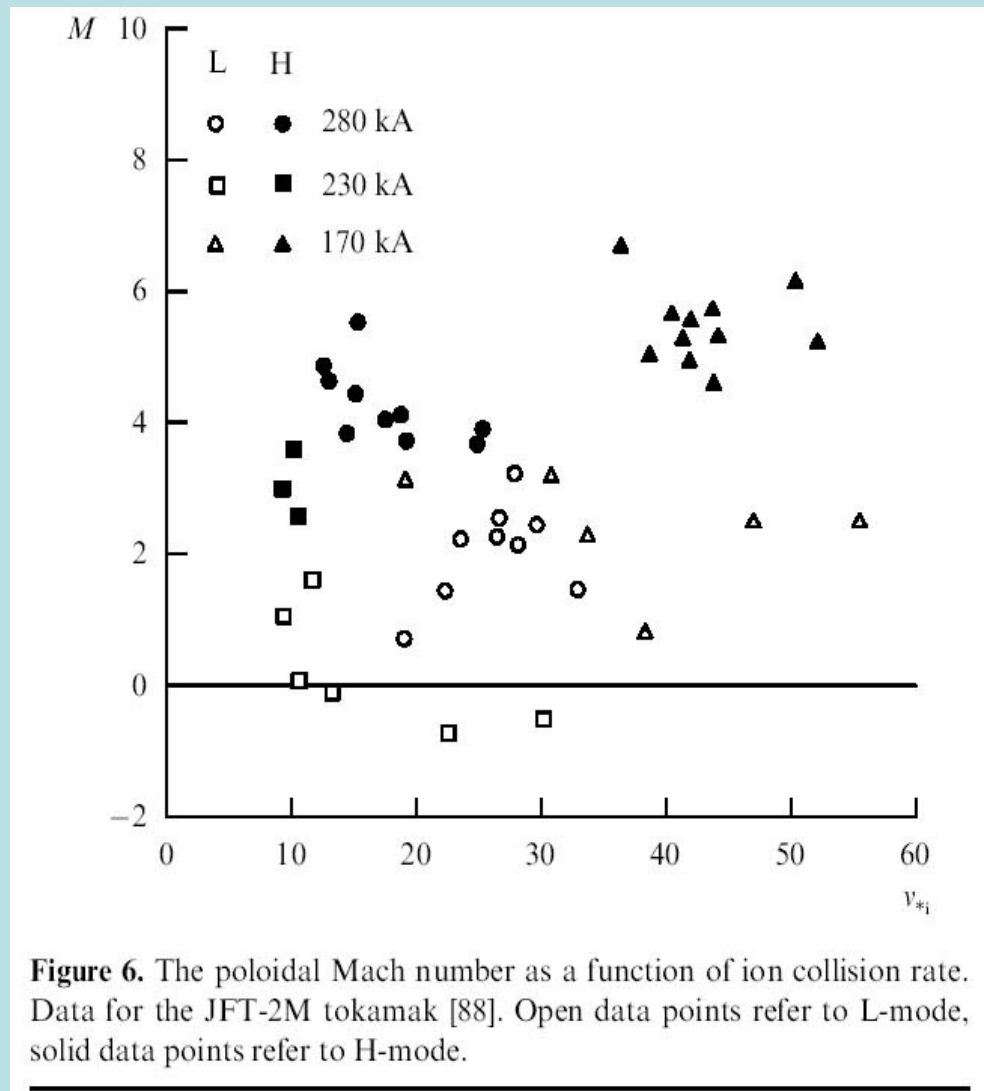
- The Poincare Invariant is a consequence of Hamiltonian nature of our world and appear in many variants of MHD and Vlasov eqs. Many models, but the same invariants and the same TEPs.
- Alternative derivation. TEP is a plateau on the distribution function of the toroidal momentum.
- Remember the quasilinear beam relaxation by Vedenov, Velikhov, Sagdeev (1962), where the incompressibility is a consequence of Liouville theorem. The only difference is dropping mechanical part of momentum  $\vec{p}=m\vec{v}+e\vec{A}/c$  instead of magnetic one.
- For simplicity consider a large aspect ratio tokamak with a circular cross-section

$$f(p)=dN/dp=const$$

$$p=eA/c \quad dA/dr=B_p \quad dN/dr=2\pi rn$$

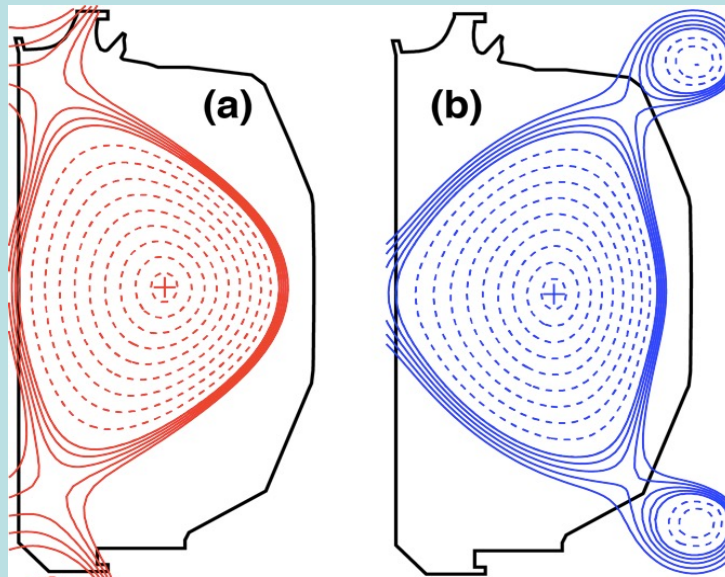
$$n(r)r=B_p(r) \quad n(r)=1/q(r)$$

Galeev, Berk and Galeev, Sagdeev, Wong predicted H-mode as an ion banana thick boundary barrier in two papers in Phys. of Plasmas in 1967!  
The cause of the barrier is loss of ions and poloidal rotation



Improvement by negative triangularity in TCV Y. Camenen et al., Nucl. Fusion **47**, 510 (2007) and DIII-D M. Austin et al., Phys. Rev. Lett. **122**, 115001 (2019) suggests a possibility operation in a highly improved L-mode

The DIII-D profiles show that placing X-points far from the center increases specific poloidal volume. I propose a third X-point in outward part of equatorial plane





Last touch: Why tokamak plasmas diffuse through the magnetic field, but the magnetic field does not? Why conductivity is close to the neoclassical one?

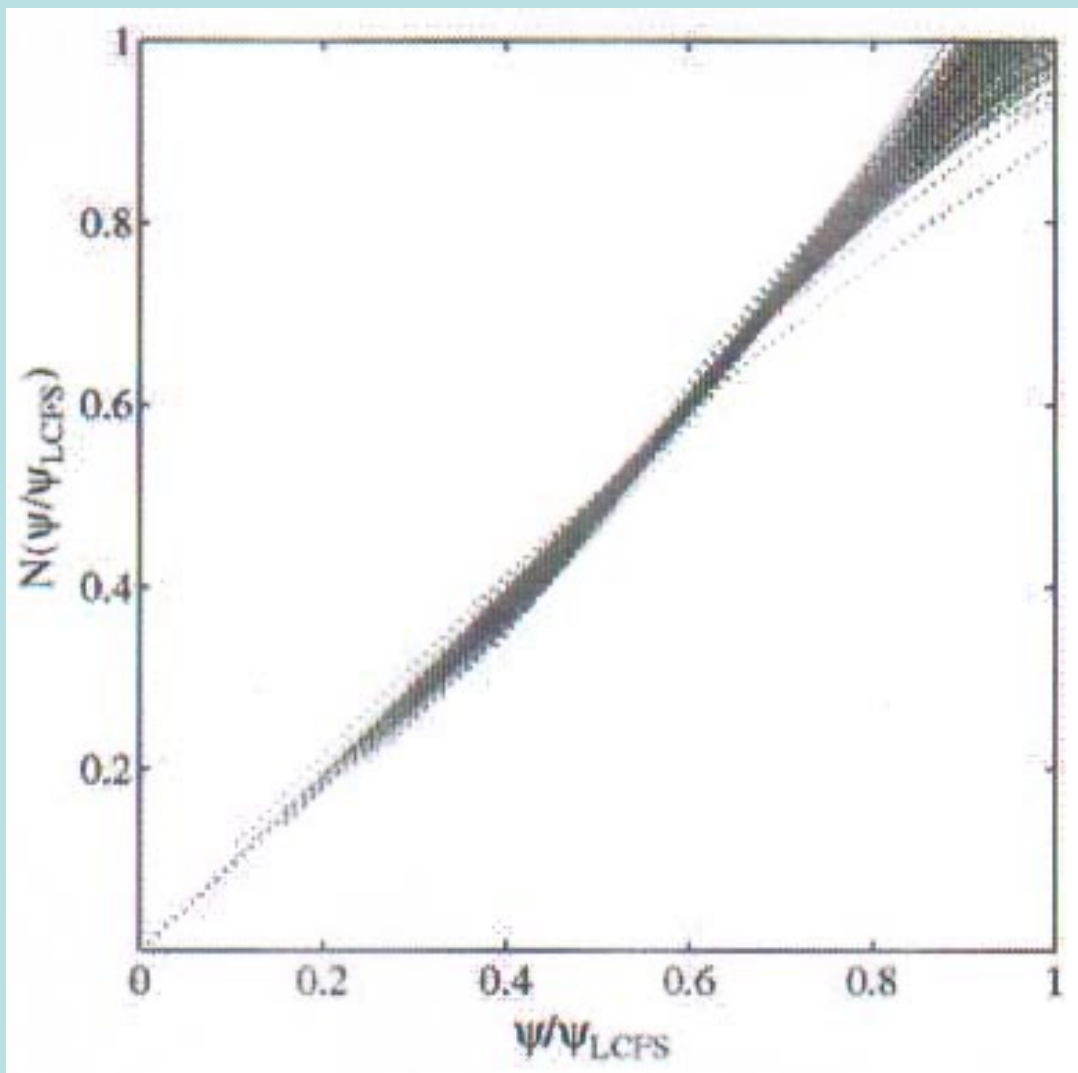
- Because passing particles are not chaotic due to KAM-like invariant tori. The passing particles work as a custodian of the topology of the magnetic field (Semi-Ideal MHD). Neglect collisions and the plasma behaves like a mixture of viscous and superconducting fluids.
- No quantum mechanics, but the fundamental reason is the same: the coexistence of integrable and chaotic particle behavior.
- Similar explanation was published by Alan Boozer *Phys. Fluids B* **2** 2300 (1990) well before me, I apologize for not mentioning him in my first publication on the subject.

# To describe TEPs in tokamaks we have to derive frozen-in laws of Vlasov eq

- Invariants appear before and are more robust than eqs of motion.
- Consider a trapped banana particle in a tokamak and assume, that turbulent perturbations are much slower than the oscillations of the particle and that banana thickness is smaller than the size of nonuniformities. It means, that we have two adiabatic invariants and motion of banana center lies in the 2d equatorial plane of a tokamak. Since only poloidal magnetic field crosses the equatorial plane, we arrive to the frozen-in law of trapped particles  **$n\mathbf{v}=\mathit{const}$** . Here the density of trapped particles is marked by the two adiabatic invariants.
- Why the whole plasma density behaves like trapped particles? Because passing particles are stable due to KAM-like invariant tori, but collisions mix them. Villagers and townspeople.
- For different frequencies of collisions and different models of turbulence many different MHDs can be used, but all MHDs will have toroidal component of the frozen-in law destroyed by the poloidal torque.

**A big problem bigger than tokamak turbulence: number of invariants is fewer than number of variables, hence our world is chaotic and nonintegrable.**

- Observation: The huge complex multidimensional world is knowable due to **simple attractors**. The process of attraction is nonintegrable, but attractors have much lower dimensions and therefore could be described analytically.
- Exact solutions are not a typical case and have a measure zero unless they are attractors.
- Ptolemy-Kepler-Newton model of Solar system is a well-known attractor, which can be described by conservation of energy, angular momentum, adiabatic invariants and KAM invariant tori.
- The Solar system is an attractor a bit more complex than energy minimum.

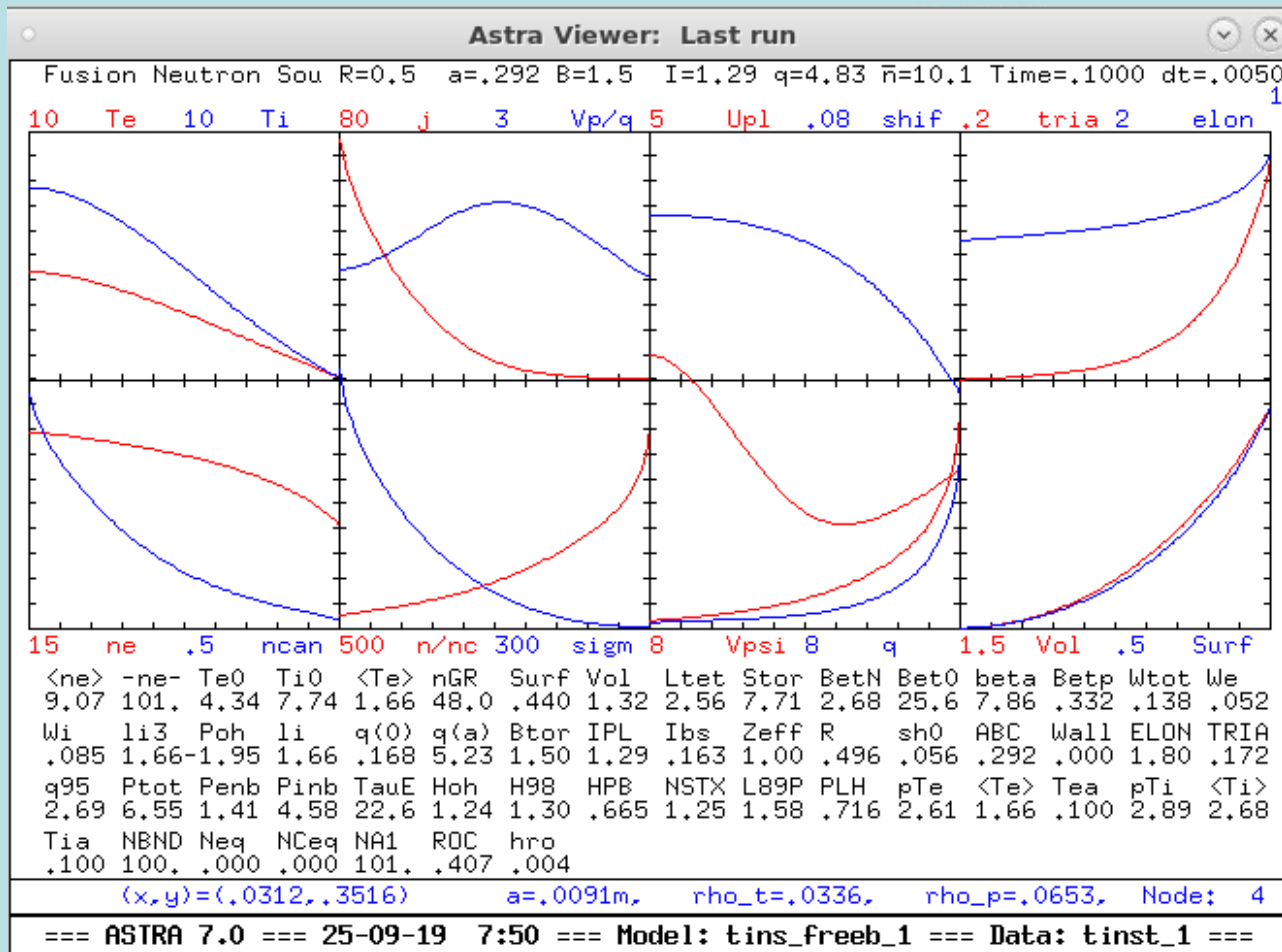


Integrated normalized particle content versus normalized poloidal flux in an ECH TCv data set. 200 density profiles from Weisen et al. 2002 Best fit 0.8 The turbulent attractor  $n \sim 1/\nu$  is confirmed and must be used to improve confinement.

# Turbulent EquiPartitions are a class of attractors

- Sugar in coffee is distributed uniformly for three reasons: conservation of sugar, water incompressibility, mixing. The concentration of sugar is constant along infinite number of trajectories  $n=const$ . The uniform distribution is an attractor  $n=const$ . Note, that the constants are different!
- Atmosphere is compressible but turbulence conserves the number of particles and the specific entropy (adiabatic law), hence, the temperature drops with altitude
- TEP of the specific entropy in the nearest thermonuclear reactor is confirmed with accuracy 99%.
- TEP was accurately derived in a magnetoelectrostatic trap from eqs of motion by V. Pastukhov Sov. Journal of Plasma Physics **6** 549 (1980).
- A TEP was assumed as a tokamak attractor, but the problem was a choice of invariants. K. Razumova gave a key hint in 1994: Enigmatic density peaking!
- Toroidal direction in tokamaks is symmetric, but poloidal one is not. Therefore, there are considerable torque forces in poloidal direction, which destroy toroidal component of frozen in law. As a result, plasma is attached to lines of the poloidal magnetic field and  $nv=const$  along trajectories like sugar.  $v$  is the volume of a poloidal magnetic tube. Assume a good mixing, and the constant will be the same inside tokamak, like sugar in coffee!

# Difference between q and v in the 7<sup>th</sup> figure, A. Dnestrovsky



# Any useful model of our complex world includes simple attractors

- I apply attractors to many problems of physics, climate, AI etc. and would like to give you several samples to promote the tool.
- TEPs are among the simplest attractors, but solitons are more complex than sugar in coffee.
- Importance of solitary waves in non-integrable systems was evident, because they appear in simulations. Theory of attractors predicted that linear waves will condensate in solitons and solitons will merge. The math obstacle was absence of an invariant measure in functional space, so the results are not rigorous.
- Simulations by A. Dyachenko, V. Zakharov, A. Pushkarev, V. Shvets, V. Yankov, JETP 1982 confirmed the predictions.

$$W = T / (f + f')$$

where  $f'$  is a negative frequency shift in solitons

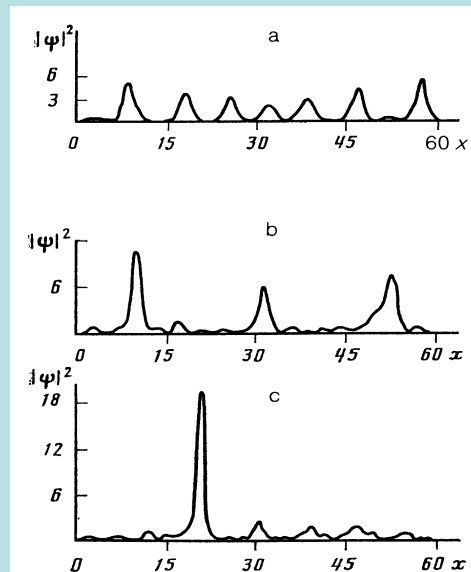
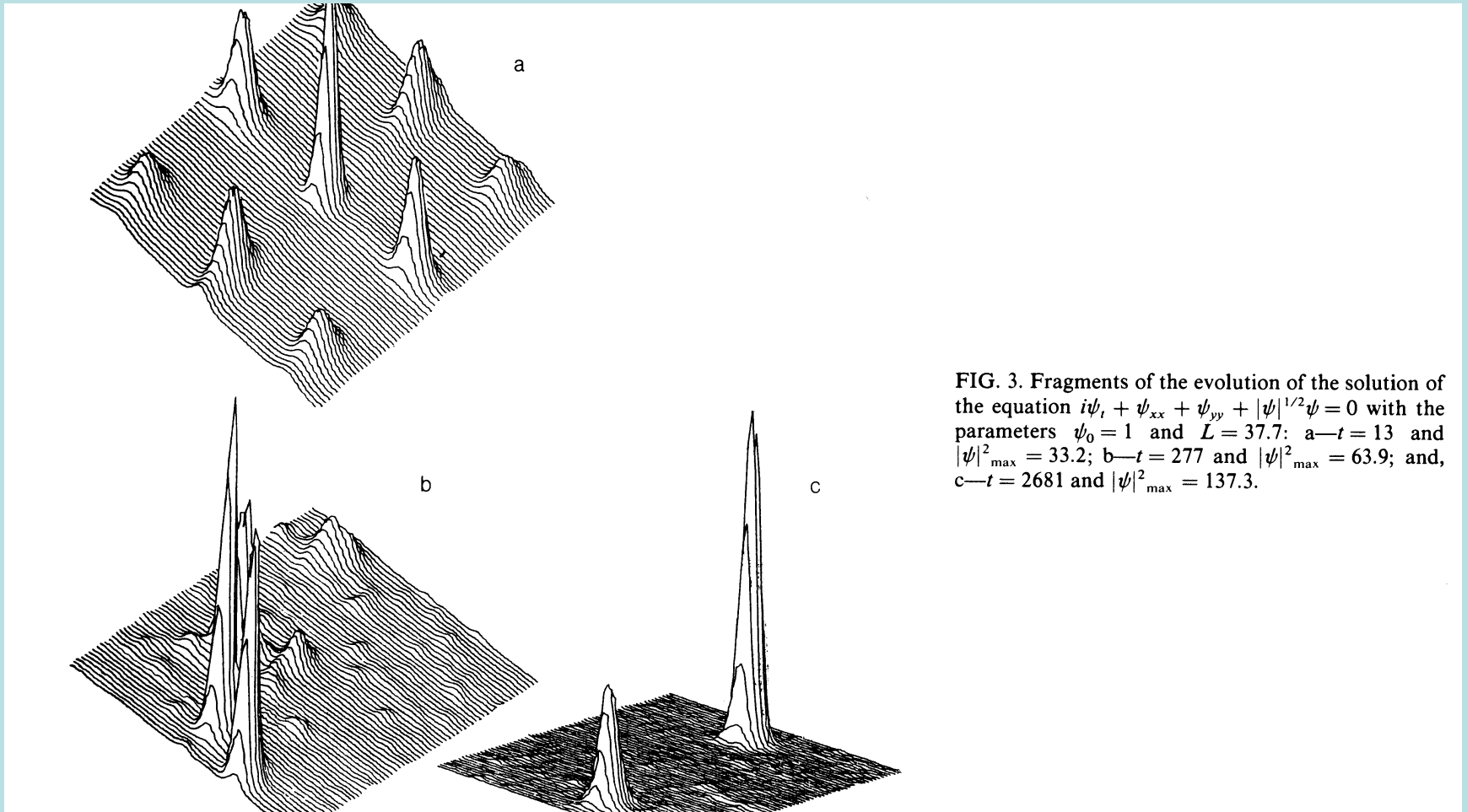


FIG. 1. Fragments of the evolution of the solution of the equation  $i\psi_t + \psi_{xx} + |\psi|\psi = 0$  with the parameters  $\psi_0 = 1$  and  $L = 60$ ; the time  $t = 17.4$  (a),  $365.4$  (b), and  $730.8$  (c).

# Condensation in 2d solitons

A. Dyachenko, V. Zakharov, A. Pushkarev, V. Shvets, V. Yankov, JETP 1982





Thin vortex attractor is defined by conservation of vorticity (Poincare invariant!) and energy maxima

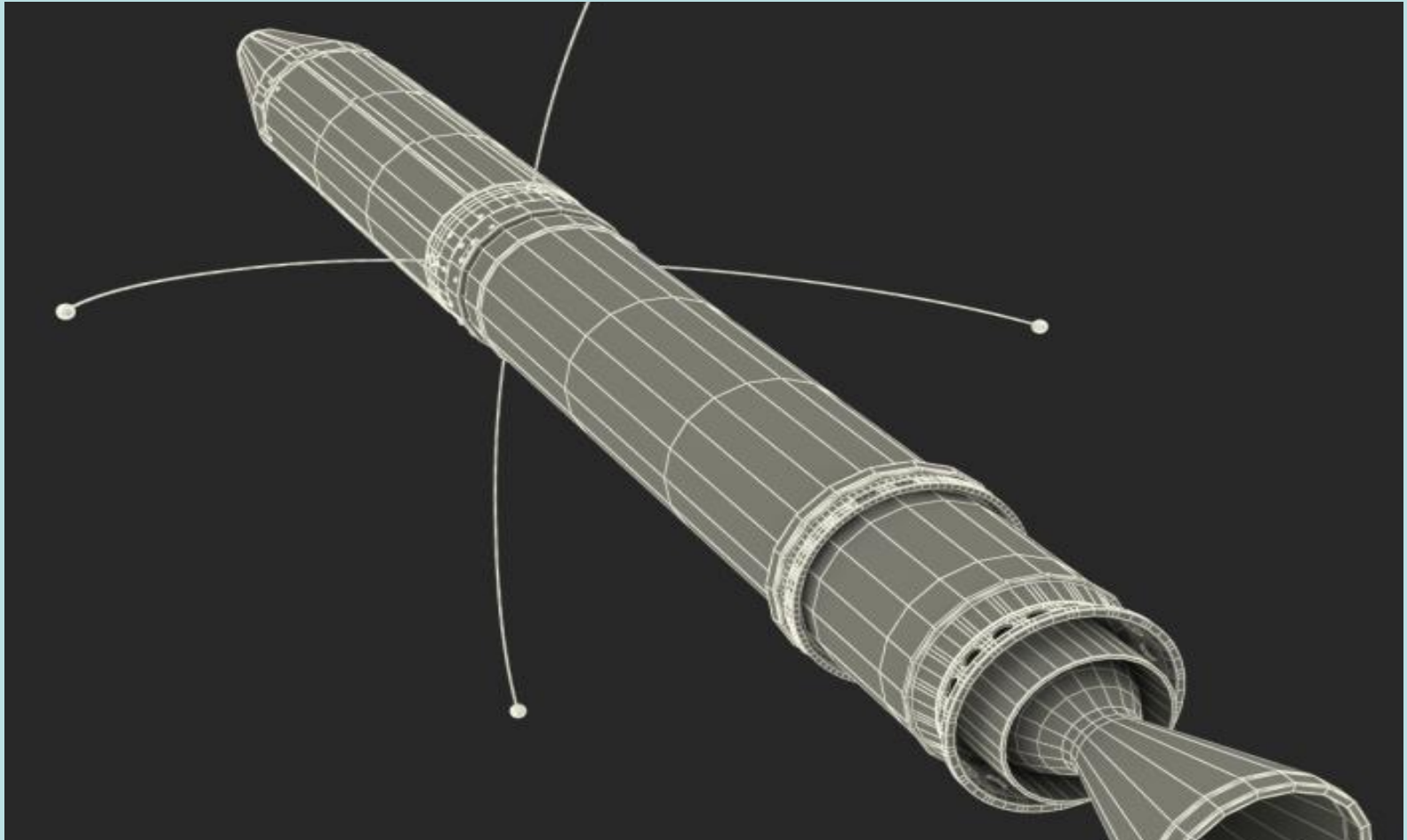


Other  
source  
of energy,  
the same  
attractor





The first satellite launched by the US was stabilized by spin like a rifle bullet, but lost orientation soon. The people used eqs of motion instead of invariants.



Oumuamua rotates chaotically hence it is not an abandon starship, which should include something flexible and rotate perpendicular to the long axis



## To predict the complex and chaotic world, including tokamaks, we must use invariants and attractors.

- The turbulent attractor  $nv=const$  is robust and has a predictive power.
- The attractor is confirmed by MHD simulations of Pastukhov, Smirnov, Chudin.
- I'm pretty sure similar attractors are hidden in gyro simulations and I'm here to help.
- The goal of my tokamak activity is to improve confinement of tokamaks in order to lower the financial barrier of ignition several times. ITER is very expensive, but hopefully not optimal.
- We must read and interpret many thousands of experimental papers, available databases, talk to experimenters to distill the data into a statement so brief, that it can be minted on a coin.

Thank you!

## Repeating objections of my friends and my answers

1. Vladimir, all your fairy tales about attractors are wrong.

Why, Boris?

I know all the textbooks by Landau and Lifshits very well. There is nothing about your attractors.

Boris is right. Books and journals are filled with theories of perturbation.

2. ITER scalings clearly indicate improvements with increase of poloidal magnetic field/toroidal current.

- ITER scalings do not incorporate current rampdown.
- The scalings show improvement with increased elongation in accordance with the suggestion of weakening.

3. Read textbooks, a magnetic field is frozen-in. Your generalized vorticity is not mentioned.

- The law was rediscovered many times, including P. Dirac (1940), S. Braginskii (1948), O. Buneman (1949), Linden-Bell (1967), V. Yankov (1979) and several times later, every time without references on previous results.

4. The Poincare Invariant and the frozen-in invariants are not related.

- The projections from 6d space to 3d space are not evident, but true.

5. There are magnetic field tubes while your poloidal magnetic tube is nonsense.

- Both are mathematical abstractions. Magnetic field tubes were drawn many thousand times in textbooks and papers, so many believe they are real.

## Repeating objections of my friends and my answers

6. With ECH, we can change density profiles as we want.
  - Attractors are elastic and overheating plasma near the center expands plasma, it's natural. Show me tokamak with maximum density near the edge where the particle source is.
7. Your attractor is just a new word with no new results.
  - In tokamaks, both the particle pinch mechanism and the profile  $n \sim 1/q$ , as well as the prediction of reversed shear stabilization, were new.
  - Experimentally the canonical profiles and the enigmatic density peaking were known long before my theory, it's true. The theory must have been made in 70ties.
8. Do you really think that theoretical physics can be done without calculations?
  - I have been doing this since my student days.
  - The outstanding Boltzmann distribution does not require calculations.

The Boltzmann attractor is an equipartition on a multidimensional hypersphere, determined by the law of conservation of energy  $E$

$$F(E) \sim (1 - E/Tn)^n = \exp(-E/T)$$

the Euler formula!



$$I = \oint \vec{p} d\vec{q}$$

**Poincare Invariant is integral form of Hamilton Equations**

$$I = \oint \vec{p} d\vec{q}$$

Adiabatic Invariant

$$I = \oint \vec{p} d\vec{q}$$

Generalized frozen-in law

$$\vec{p} = m\vec{v} + e\vec{A}/c$$

# Reversed shear stabilization has been predicted as energy minimum

«Suppression of turbulence can be obtained without moving the heat source in the atmosphere, but by inverting gravity. In tokamak, a similar effect is achieved by decreasing the safety factor  $q$  with a radius...» Yankov, Soviet Fizika Plazmy (1995)

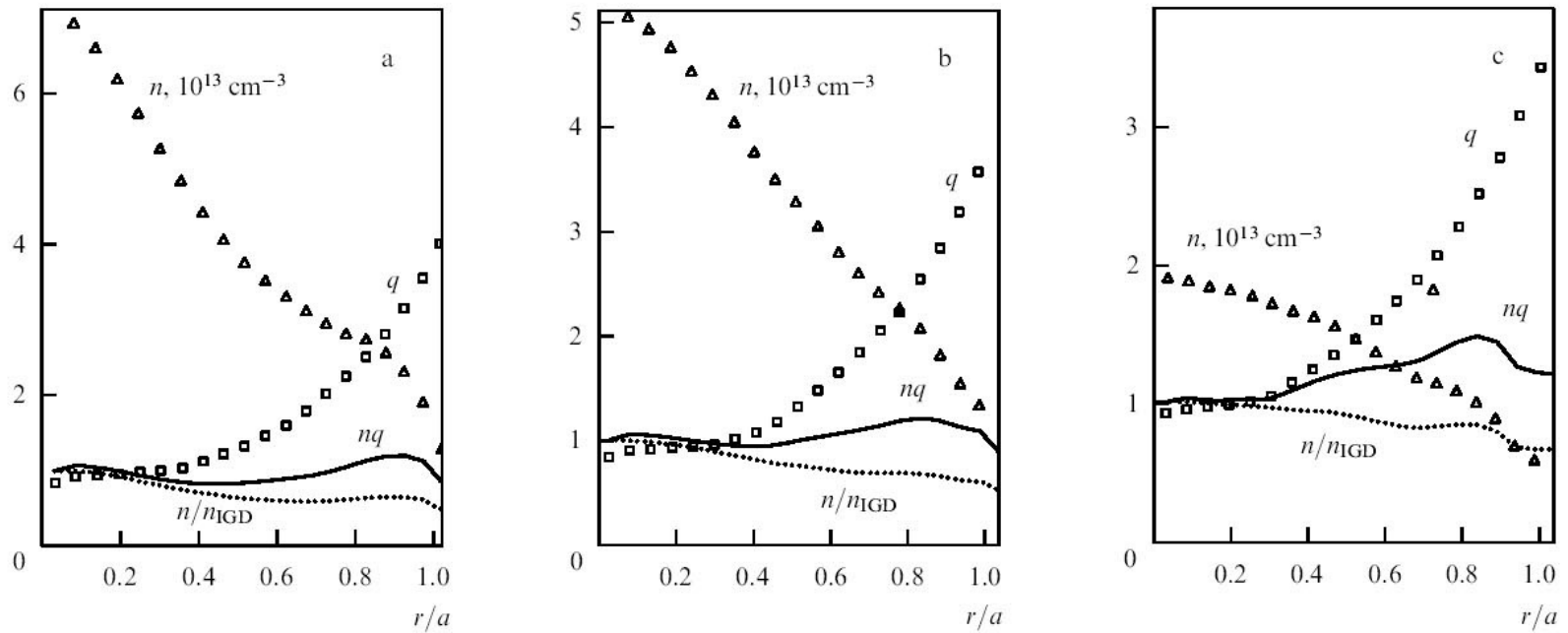
$$dq/dr < 0$$

The theory has predictive force! It was suggested a half year before discovery by TFTR Levinton et al, DIII-D Strait et al PRL (1995).

As shear flattens, rational surface barriers appear and merge if the shear is negative

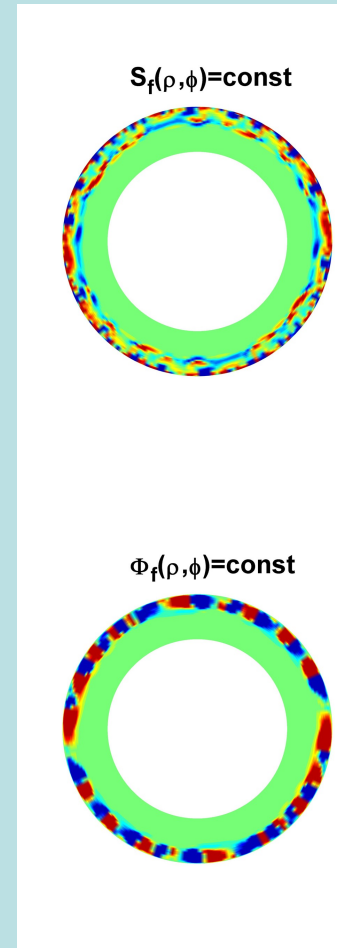
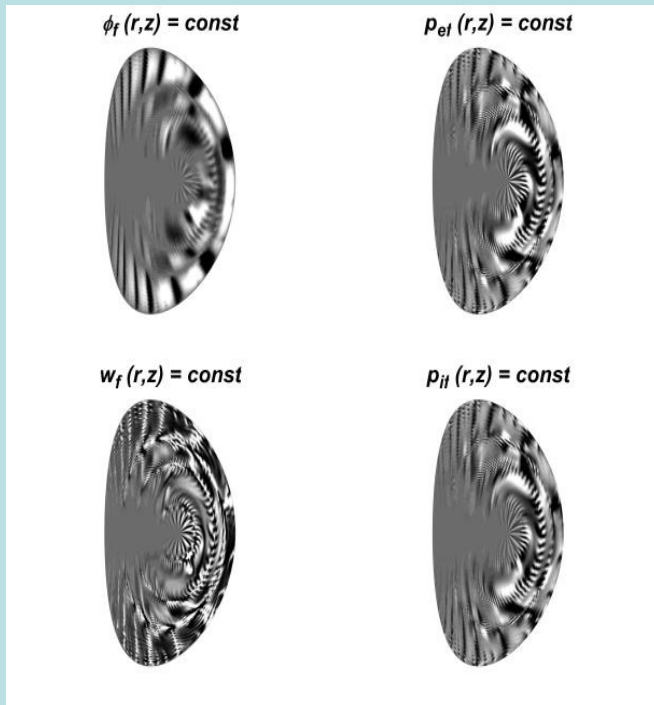
**Tokamak profiles are hierarchy of three attractors. The first is a simple minimum of magnetic energy as was explained by Kruskal and Shafranov, the second one is Turbulent EquiPartition of angular magnetic momentum, the third is current profile**

- Experimenters observed similar profiles of plasma in tokamaks beginning 1962. Canonical profiles of pressure were described by Coppi and later by Esipchuk and Razumova.
- The puzzling feature of experimental profiles is a maximum of plasma density in center and minimum at boundary, while the source of particles is at the boundary! Central maximum is natural for temperature because the plasma is heated at the center.
- The hint: what set of invariants leads to a turbulent attractor with maximum at the center?
- The hunt has started from Poincare invariant application to drift Vlasov equations, which resulted in new set of frozen-in invariants, relatives of frozen-in invariants of magnetic field and adiabatic invariants.
- Finally, the angular magnetic momentum leads to the central maximum of density



**Figure 4.** Profiles of the density  $n$ , the safety factor  $q$ , and the Lagrangian invariant  $nq$  in the TFTR tokamak [69]: (a) supershot 76770, temperature peak; (b) the same supershot after switching-off the heating before crash; (c) L-mode. Here,  $n_{IGD}$  is the profile suggested in [77].

# MHD simulations by Pastukhov, Chudin, Smirnov, Dnestrovsky confirmed TEPs and ITBs



**The chaotic and complex world is knowable and predictable due to attractors; the success is due to reduction of number of parameters around  $10^{30}$**

Samples of attractors: energy minima and maxima, Boltzmann distribution, solitons and wave collapses, vortices, measurements in quantum mechanics, brain and AI, Life, z-pinch singularity, turbulent attractor in tokamaks (Yankov UFN, 1997)

The best way to study attractors is simulations  
What the Amish say about science? Too wordy.